

Review Problems for Final Exam

These review problems cover the material in Lessons 11-14. Take a look at the review problems for Exam 1 and Exam 2 as well.

Problem 1. Customers arrive at Fantastic Dan's hair salon according to a Poisson process at a rate of 5 customers per hour. There are only 3 chairs provided for customers to wait for Dan. The time for Fantastic Dan to cut one customer's hair is exponentially distributed with a mean of 10 minutes. Since Fantastic Dan's customers are rather impatient, they may renege: the time a customer is willing to spend waiting before starting his or her haircut is exponentially distributed with a mean of 15 minutes.

- a. Model this queueing system as a birth-death process by defining
 - the state space and what each state means,
 - the arrival rate in each state, and
 - the service rate in each state.
- b. Over the long run, what is the expected number of customers in the hair salon?
- c. Over the long run, what is the expected time a customer spends in the hair salon?

Problem 2. The Simplexville Community Center has 5 tennis courts. Pairs of players arrive at the courts according to a Poisson process with rate of one pair per 10 minutes, and use a court for an exponentially distributed time with mean 40 minutes.

- a. What standard queueing model fits this setting best?
- b. What is the long-run expected fraction of time that all courts are empty?
- c. Over the long run, how many pairs of players are in the queue to get a court on average?
- d. What is the long-run expected delay time to get a court?
- e. Suppose the playing times are actually uniformly distributed between 30 and 50 minutes. Give an approximation of the expected delay time to get a court.

Problem 3. You have been put in charge of redesigning Turingtown's emergency call center. Based on historical data, you have estimated that calls arrive at a rate of 12 per hour, and that each call takes 1 operator 15 minutes on average. If all operators are busy when a call arrives, that call is put on hold. Model the call center as an $M/M/\infty$ queue and determine the minimum number of operators needed to ensure that no calls are put on hold 99% of the time in the long run.